MATH 317, Summer 2016 Syllabus

Course Title: Advanced Calculus of One Variable (i.e., "Real Analysis")

Time/Location: MTWRF, 2:30-3:50 pm, Engineering B105. Office hours TBD

Instructor: Vance Blankers, blankers@math.colostate.edu

Textbook: *Elmentary Analysis: The Theory of Calculus* - 2nd Edition, Kenneth Ross

Content: From the course catalogue: "Convergence of sequences, series; limits, continuity, differentiation, integration of one-variable functions; development of skills for proving theorems." In particular, we will prove and better understand most of the theorems used in your intro calculus course, developing mathematical intuition and the ability perform rigorous mathematical reasoning along the way.

Grading: Letter grades will correspond to 10% windows: 90-100% is an A, 80-89% is a B, etc. The following items will contribute to your final grade (40% homework and quizzes, 60% exams):

> Homework - Solutions will be graded on correctness and clarity of supporting work. Having the right answer or right idea is *not* enough for full credit; you must be clear in your communication as well.
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> Quizzes - About once a week we will have quizzes, for which you will be asked to write precise definitions and/or statements of important theorems.
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> Exams - We will have three exams, including the final. They will be a fusion of homework-style questions and quiz-style questions; this does not mean they will be comprised only of previously-seen questions.

Academic Integrity: Don't cheat. Check out `http://tilt.colostate.edu/integrity` for more details. While many things in life operate on the "better to ask forgiveness than permission" principle, this is not one of them. When in doubt, ask me ahead of time. Googling (or Binging or whatever) the answer to an assigned question is absolutely cheating, regardless of how you try to justify it to yourself.

Groupwork, unless specified otherwise, is *not* considered cheating in this class, and is very strongly *encouraged*. However, you are expected to write up your solutions individually; word-for-word reproductions look fishy at best, so please make sure to write things in your own words.

RDS: Have a Resources for Disabled Students (RDS) situation? No problem; just let me know as soon as possible.

Late Homework: In general, no late work will be accepted. You'll be asked to turn in homework at the beginning of class on whichever day it is due, though you can always turn it in early. Exceptions for extreme circumstances and emergencies, accompanied by written documentation of proof, will be considered but not guaranteed.

Scheduling Conflicts: Let me know as soon as possible if you have some sort of conflict with the class schedule. I'm happy to make reasonable accommodations, but only if I know ahead of time.

Other Expectations: Treat your classmates and me with respect: silence cell phones when you get to class, don't cause distractions during lecture, don't eat delicious-smelling food without sharing, etc. Homework must be written legibly and separate sheets stapled, with no fringes; points will be docked for failing to meet these requirements. If your handwriting is atrocious, either practice or type up your solutions. Finally, I expect you to give an honest effort and have a good attitude. The number one cause of poor performance in a math class is an "I can't do it" mentality.

Leftovers: Extra stuff that didn't fit any of the categories above:

> As the instructor, I reserve the right to alter this syllabus at any time. I'll announce any such changes in class, in as timely a manner as possible.
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> If you have any issues at all, please do not hesitate to contact me. Most problems can be resolved via communication.
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> This is a summer course, so it's condensed and quick-paced. *Do not get behind.* This class will require a significant chunk of out-of-class time; make sure you respect the amount of work needed.
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> Technology is a double-edged sword in learning mathematics. You should attempt to use technology to enhance your understanding without using it as a crutch. For example, an appropriate use of technology is to look up terminology that is new to you.
>
> Related to the above, patience is your biggest ally. You will get stumped from time to time. Resist the urge to immediately ask for help or to right away Google the answer. Instead, try different things; see what you can do with the tools given. Draw a picture. Attempt to do the stupidest, most straight-forward thing possible, and work from there. The process of exploring questions and actively struggling with them will be the most helpful aspect of the class. Don't be Flanders Sr.:

Useful Symbols: Since I can't read your mind, your ability to communicate your understanding of course material is just as important as your actual understanding of it. Throughout the course, you should strive to be as precise and accurate as possible when trying to state and justify mathematical ideas – to this end, the following symbols should be a valuable tool. Keep in mind that using a symbol *does not* obviate the need for good grammar; your "mathematical sentences" should be valid English sentences as well.

- $\forall$ "for all" or "for every" [note: this is called the "universal quantifier"]
- $\exists$ "there exists (at least one)" [note: this is called the "existential quantifier"]
- ! "unique" or "exactly one"
- $\neg$ logical negation
- : or s.t. "such that" or "so that"
- $\cup$ "union" (logical "or")
- $\cap$ "intersection" (logical "and")
- $\Rightarrow$ "implies"
- $\ldots$ or $\cdots$ "etc." or "more of the pattern that was just given"
- $\{\ldots\}$ a set
- $\in$ "(is) contained in" or "(is) a member of" or "(is) in"
- $\subset$ (resp. $\subseteq$) "(is a) proper subset (of)" (resp. "(is a) subset (of)")
- | "such that" or "so that" in set-builder notation (can be replaced by ":")
- Q.E.D. or $\square$ "*quod erat demonstrandum*," Latin for "which is what had to be proven"; used to denote the end of a proof
- all of the usual math symbols like $+, -, \cdot, \geq$, etc.

Here are some examples; the "translations" into English aren't necessarily word-for-word, but each communicates a logically equivalent idea.

- $\forall n \in \mathbb{N}, \exists m \in \mathbb{N} : m > n$. "For every natural number n there exists (at least one) natural number m so that m is greater than n."
- $a \in \{x \in \mathbb{R} \mid \exists y \in \mathbb{R} \text{ s.t. } y^2 = x\} \Rightarrow a \geq 0$. "If a is a member of the set of real numbers which are the square of another real number, then a is non-negative."
- $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$. "Some complex numbers are real, and some of those are rational, and some of those are integers, and some of those are positive."
- $\exists! \, a \in \mathbb{R}$ s.t. $a^2 = 0$. "There is exactly one real number which scares to zero."

These examples had both a symbolic version and an English version; in practice we almost always use a combination of both. For example, here is one way to state the Mean Value Theorem:

- Let $a, b \in \mathbb{R}$ with $a < b$. If a real-valued function f is continuous on $[a, b]$ and differentiable on (a, b), then $\exists \, c \in (a, b)$ such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.

We will introduce additional notation if necessary, but these will cover most of our needs.

Tentative Daily Schedule

Date	Topic
6/13	Sets
6/14	Relations and Functions
6/15	Mathematical Induction
6/16	Equivalent and Countable Sets
6/17	Real Numbers
6/20	Completeness Property of $\mathbb{R}$
6/21	Sequences and Convergence
6/22	Arithmetic Operations on Sequences
6/23	Subsequences and Monotone Sequences
6/24	Cauchy Sequences
6/27	Definition of the Limit (of a Function)
6/28	Sequential Characterization of the Limit
6/29	Algebra of Limits
6/30	Infinite Limits and Limits at Infinity
7/1	Limits of Monotone Functions
7/4	No Class – Holiday
7/5	Exam 1
7/6	Continuity of a Function at a Point
7/7	Algebra of Continuous Functions
7/8	Real Line Topology

Date	Topic
7/11	Uniform Continuity
7/12	Properties of Continuous Functions
7/13	The Derivative of a Function
7/14	Algebra of Derivatives
7/15	Rolle's Theorem, Mean Value Theorem
7/18	Taylor's Formula
7/19	L'Hôpital's Rule, Inverse Function Theorem
7/20	Exam 2
7/21	The Riemann Integral
7/22	Classes of Integrable Functions
7/25	Riemann Sums
7/26	The Fundamental Theorem of Calculus
7/27	Algebra of Integrable Functions
7/28	Derivatives of Integrals
7/29	Mean Value, Change of Variable Theorems
8/1	Convergence of Infinite Series
8/2	Absolute Convergence, Comparison Test
8/3	Ratio and Root Tests
8/4	Buffer
8/5	Final Exam