MATH 3175, Fall 2020 Syllabus

Course Title: Group Theory

Time/Location: MWR, 10:30–11:35 am, Richards Hall 325 (remotely via Zoom)

Office Hours: M 12 – 2pm, T 10am – 12pm, W 2 – 4pm, via Zoom

Instructor: Vance Blankers, `v.blankers@northeastern.edu`

Textbook (optional): *Abstract Algebra*, J. Beachy & W. Blair, (any edition)

Content: This course presents the basic concepts and techniques of group theory. We plan to cover the following topics: a primer on set theory and binary operations, the axiomatic definition of groups, important classes of groups, subgroups, group actions, direct products, homomorphisms, quotient groups, conjugacy, and combinatorial applications of group theory.

Grading: The following items will contribute to your final grade.

- Homework (35%)
- Midterm 1 (15%) – due October 5
- Midterm 2 (15%) – due November 9
- Final exam (25%) – due December 15
- Attendance and participation (10%)

Course Format: Our class will meet remotely, via the Zoom link on Canvas. Here's how that'll work:

- Lectures will be recorded, and recordings will be posted on Canvas by the end of the same day. However:
- You are expected to attend lectures **at the class time**, provided you are able to do so. If you are in a time zone such that you cannot attend lectures, or if you miss a lecture, you must watch the recording and send me a 5-sentence summary of what was covered within 48 hours of the end of the missed lecture; this will count towards your attendance/participation grade.
- In general, try to keep your **video on** and **microphone off** during lectures. You can also unmute to ask a question if that's easier than asking in the chat.
- During each lecture, one student will be assigned (or someone can volunteer!) to monitor the Zoom chat in order to bring questions and comments to my attention that I might accidentally miss.
- Lecture recordings will only be distributed to your classmates. If you are unwilling to be recorded, let me know by email, and make sure you have your video off during lectures in this case.
- All three exams will be take-homes; more details will be available when we get closer to the dates.

Homework: Homework occupies the largest chunk of your overall grade. More about that:

- Homework will be assigned on Fridays (posted on Canvas) and will be due (submitted via Canvas) the following Thursday by 11:59pm Eastern.
- Late homework will not be accepted unless you have made prior arrangements with me.
- The lowest homework score will be dropped. No other curve is guaranteed.
- In contrast to most math classes, collaboration is **discouraged** in this course. Contributing equally on proofs is incredibly difficult, and you will learn the material far better if you struggle through it solo. **The goal is to understand and master the material, not finish any given assignment as quickly as possible.** If you do work with someone on a problem, include that person's name next to that problem on your submitted solutions, and make sure you write things up on your own.
- Don't cheat. While many things in life operate on the "better to ask forgiveness than permission" principle, this is not one of them. When in doubt, ask me ahead of time.
- Solutions should be presented legibly and professionally. If your handwriting is atrocious, either practice or type up your solutions. This is a great time to learn LaTeXif you haven't yet!

Advice for this course: This course will likely be quite different from other math courses you've taken; in particular, this course is predominantly proof-based. Here are some pieces of advice:

- Watching Khan Academy or similar videos is usually unhelpful and often counterproductive for learning how to write proofs.
- Memorizing the proofs of theorems is usually not helpful. Memorizing definitions is **very** helpful, to the point of being borderline mandatory.
- Almost any problem assigned in this course can be found online with some clever searching; this is a universally bad idea. Looking up a solution "just to check your work" or "just to get an idea of how to do it" never works to aid learning, despite how tempting it may sound.
- As some extra incentive to avoid Google: I am very, very good at searching for math stuff online. If you manage to find a solution to some homework problem online, there is very little chance that I haven't also found that solution while writing the problem.
- Your goal should be to **do** math, not know it. The process of learning math is very similar to the process of learning basketball (or any other skill); you don't get better by watching, only by practicing.
- Patience is your biggest ally. You will get stumped from time to time. Resist the urge to immediately ask for help or to right away Google the answer. Instead, try different things; see what you can do with the tools and techniques you have. Draw a picture. Attempt to do the stupidest, most straight-forward thing possible, and work from there.
- Become comfortable with discomfort and struggle. You should be wrong **a lot** before you're right, and that's okay. You'll hear me say the phrase "conservation of misery" quite often; try to embrace this concept!
- See the next page for some specific ideas of how to proceed when stuck on a proof.

Leftovers: Extra stuff that didn't fit any of the categories above:

- As the instructor, I reserve the right to alter this syllabus at any time. I'll announce any such changes in as timely a manner as possible.
- Every student is expected to complete the online TRACE survey at the end of the semester.
- If you have any issues at all, please do not hesitate to contact me. Pretty much every (non-homework) problem can be resolved via communication. If you do not feel comfortable talking to me directly, you are able to contact the Teaching Director, Professor Alex Martsinkovsky (`a.martinskovsky@northeastern.edu`).
- This is a fast-paced course. **Do not get behind.** This class will require a significant chunk of out-of-class time; make sure you respect the amount of work needed.
- That said, I know you have multiple classes this semester (and I know you have a life outside of classes as well!). You're human – and so am I – so please let me know when things are getting overly busy or stressful.

When you're stuck on a proof, try the following

- Don't wait to the last minute to churn out a proof. You will often be better served by working on it for a while, then stepping away and working on something else (or going for a walk, grabbing food, etc.). Coming back later often makes things "click" much easier. This is probably the most useful bit of advice.
- Make sure you know the definitions for all of the terms appearing in the statement you're trying to prove. For example, in the statement "If the order of all nontrivial elements in a group is two, then the group is abelian," the words "order," "nontrivial," "elements," "group," and "abelian" all have specific, group-theoretic definitions that are important in proving the statement.
- Naming things is a powerful tool. If we have "a group with three elements," we might say that our group is G; since G is a group, it has an identity we call e; with the two other elements, we might write $G = \{e, a, b\}$. Our brains like names, and they often make things more workable.
- Abstraction is important, but it can be helpful (sometimes necessary!) to make things more concrete (in particular by giving things names).
- Be careful about assumptions. If we are trying to prove something about all groups, we cannot start with "Assume G is abelian," since not all groups are abelian. On the other hand, if you are given the assumption in the statement, you probably need to use it! So if we're trying to prove that all abelian groups have some property, we probably need to use the definition of "abelian" at some point in the proof.
- Write out explicit examples. **Examples are not proofs**, but they can give hints on how to make more abstract arguments. In particular, small examples (even trivial examples!) can be immensely helpful.
- The advice "Make things concrete" is different than relying on explicit examples. E.g., if we have two subgroups H_1 and H_2 of a group G which intersect non-trivially, we may say something like "There exists an element $h \in G$ such that $h \neq e$, $h \in H_1$, and $h \in H_2$." This makes the intersection **concrete**, and we are free to work with h while writing our proof. In contrast, if we tried to say "Let $G = \{e, a, b, c, d\}$, $H_1 = \{e, a, b, c\}$, and $H_2 = \{e, a\}$," then we are making an **example**, which we cannot directly use in our full proof (though it can give us hints at how to make an abstract proof).